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Phase-space study of a spinning quantum particle in a constant magnetic field

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Abstract. Landau levels are obtained using the techniques of phase-space quantum mechanics. Wigner functions are calculated for the eigenstates of this system.

In some recently published papers a new basis [1–5] for the classical formalism of quantum mechanics in phase-space has been established [6–9]. The original limitation of the formalism to spinless particles has been overcome [2, 10]. However, the correspondence between quantum spin states and observables and functions on the sphere analysed in these papers is not directly used to solve any problem of physical interest. Here we apply the new spin formalism to a very well known problem: a spinning particle immersed in a constant magnetic field.

The relevant phase-space is $\mathbb{R}^6 \times S^2$, with coordinates $\gamma \equiv (\boldsymbol{q}, \boldsymbol{p}; \boldsymbol{n}) \equiv (\boldsymbol{u}; \boldsymbol{n})$, $\boldsymbol{n}$ being the coordinates of a point on the sphere S^2. The non-commutativity of the operator product in quantum mechanics translates into a non-commutative operation between functions defined on a phase-space, the so-called twisted product. In this case, it is given by

$$(f \times g)(\gamma) = \int_{\mathbb{R}^6 \times S^2} \int_{\mathbb{R}^6 \times S^2} f(\gamma')g(\gamma'')\mathcal{L}(\gamma, \gamma', \gamma'')\, \mathrm{d}\gamma'\, \mathrm{d}\gamma'' \tag{1}$$

where f, g are functions defined over the phase-space, the measure over this space is $\mathrm{d}\gamma = \mathrm{d}\boldsymbol{u}\, \mathrm{d}\boldsymbol{n} = \mathrm{d}q_1\, \mathrm{d}q_2\, \mathrm{d}q_3\, \mathrm{d}p_1\, \mathrm{d}p_2\, \mathrm{d}p_3 \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\phi$, (θ, ϕ) being the usual spherical coordinates, and the integral kernel $\mathcal{L}$ is

$$\mathcal{L}(\gamma, \gamma', \gamma'') = L(\boldsymbol{u}, \boldsymbol{u}', \boldsymbol{u}'')\; L^{1/2}(\boldsymbol{n}, \boldsymbol{n}', \boldsymbol{n}'') = \left(\frac{1}{\pi}\right)^6 \exp\left[2\mathrm{i}(\boldsymbol{u}^t \mathbf{J} \boldsymbol{u}' + \boldsymbol{u}'^t \mathbf{J} \boldsymbol{u}'' + \boldsymbol{u}''^t \mathbf{J} \boldsymbol{u})\right]$$
$$\times \left(\frac{1}{4\pi}\right)^2 \{1 + 3(\boldsymbol{n}\cdot\boldsymbol{n}' + \boldsymbol{n}'\cdot\boldsymbol{n}'' + \boldsymbol{n}''\cdot\boldsymbol{n}) + \mathrm{i}3\sqrt{3}[\boldsymbol{n}, \boldsymbol{n}', \boldsymbol{n}'']\}. \tag{2}$$

By $\mathbf{J}$ we represent the matrix

$$\mathbf{J} = \begin{pmatrix} 0 & \mathbf{I} \\ -\mathbf{I} & 0 \end{pmatrix} \tag{3}$$

$\mathbf{I}$ being the 3×3 identity matrix. Units are taken throughout so that $\hbar = c = 1$.

It is very important that the spin and spatial variables appear uncoupled in the kernel so that we can study systems in which we consider either the spin variables [2] or the orbital ones only [11], by simply taking $L^{1/2}(\boldsymbol{n},\boldsymbol{n}',\boldsymbol{n}'')$ or $L(\boldsymbol{u},\boldsymbol{u}',\boldsymbol{u}'')$ as the kernel respectively.

If $H(\gamma)$ is a classical (time-independent) Hamiltonian, we can obtain an operator $\mathcal{H}$ associated with it using the Weyl correspondence. If H is real, we know that $\mathcal{H}$ will be symmetric. Let us suppose that it is, in fact, self-adjoint. We can define the unitary evolution operator

$$U(t) = \exp\{-\mathrm{i}t\mathcal{H}\}. \tag{4}$$

Its phase-space counterpart, the *Moyal propagator*, is given by

$$\Xi_H(\gamma;t) = 1 + \frac{1}{1!}(-\mathrm{i}t)H + \frac{1}{2!}(-\mathrm{i}t)^2 H \times H + \frac{1}{3!}(-\mathrm{i}t)^3 H \times H \times H + \cdots. \tag{5}$$

The Fourier transform with respect to time of this function provides us with the spectral projector:

$$P_H(\gamma;E) = \frac{1}{2\pi}\int_{\mathbb{R}} \Xi_H(\gamma;t)\mathrm{e}^{\mathrm{i}tE}\,\mathrm{d}t. \tag{6}$$

It has been proved [11] that the spectrum of $\mathcal{H}$ is the support on the variable E of $P_H(\gamma;E)$. Therefore, we have an alternative way of evaluating the spectrum of a quantum mechanical operator.

Let us write down the relevant Hamiltonian. The free Hamiltonian for a spinning particle can be written $(1/2m)(\boldsymbol{p}\cdot\boldsymbol{W}) \times (\boldsymbol{p}\cdot\boldsymbol{W})$, where $\boldsymbol{W}(\boldsymbol{n}) = (\sqrt{3}/2)\boldsymbol{n}$ denotes the vector function associated with the vector spin operator $\boldsymbol{S}$. Using the minimal coupling recipe to introduce the electromagnetic interaction in the phase-space formalism [2], we get

$$H = \frac{2}{m}[(\boldsymbol{p}-e\boldsymbol{A})\cdot\boldsymbol{W}] \times [(\boldsymbol{p}-e\boldsymbol{A})\cdot\boldsymbol{W}] + e\Phi \tag{7}$$

where $\boldsymbol{A}(\boldsymbol{q},t)$ and $\Phi(\boldsymbol{q},t)$ are the vector and scalar potentials respectively. Taking into account that $\boldsymbol{A}$ is independent of the momentum coordinates, it is easy to see that

$$H = \frac{1}{2m}(\boldsymbol{p}-e\boldsymbol{A}) \times (\boldsymbol{p}-e\boldsymbol{A}) - \frac{e}{m}\boldsymbol{B}\cdot\boldsymbol{W} + e\Phi \tag{8}$$

with $\boldsymbol{B} = \mathrm{rot}\boldsymbol{A}$. In our formalism equation (8) is equivalent to the Pauli equation. After an elementary calculation, we can convince ourselves that it is possible to substitute the twisted product in (8) with the ordinary scalar product, so that

$$H = \frac{1}{2m}(\boldsymbol{p}-e\boldsymbol{A}(\boldsymbol{q}))^2 - \frac{e}{m}\boldsymbol{B}(\boldsymbol{q})\cdot\boldsymbol{W}(\boldsymbol{n}) + e\Phi(\boldsymbol{q}). \tag{9}$$

We want a constant magnetic field $\boldsymbol{B}$; let us choose it in the direction of the positive z-axis, $\boldsymbol{B} = B\boldsymbol{k}$. Here, we use the gauge $\boldsymbol{A} = (-Bq_2, 0, 0)$. It is important to emphasize that some of the results are gauge-dependent (in particular Wigner functions). The gauge we have chosen is the most suitable for easing the work (it is, by the way, the one chosen by Landau [12]). Therefore, we can write (9) as

$$H = \frac{1}{2m}\boldsymbol{u}^t \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (eB)^2 & 0 & (eB) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (eB) & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \boldsymbol{u} - \frac{eB}{m} W_z(\boldsymbol{n}) = h(\boldsymbol{u}) + s(\boldsymbol{n}). \tag{10}$$

In (10), the spatial and spin variables are separated. This point is very important in the following. We want to evaluate the Moyal propagator $\Xi(\gamma; t)$ associated with the Hamiltonian in (10), using (5). Then we will evaluate the spectral projector (6) and finally the spectrum of the Hamiltonian, analysing the support on E of the spectral projector. In order to calculate (5), we use the twisted product given by (1) and (2), where again the spatial and spin variables are not mixed. Due to these facts, it is trivial to verify that the complete propagator factorizes according to

$$\Xi_H(\boldsymbol{u}, \boldsymbol{n}; t) = \Xi_h(\boldsymbol{u}; t) \times \Xi_s(\boldsymbol{n}; t) = \Xi_h(\boldsymbol{u}; t)\Xi_s(\boldsymbol{n}; t). \tag{11}$$

The Hamiltonian of a spinning particle in a constant magnetic field belongs to the class of *distinguished* Hamiltonians, for which the Moyal propagator can be computed in closed form. The spin part turns out to be easier. Introducing the cyclotronic frequency, we have $s(\boldsymbol{n}) = \omega W_z$. Using (5), we get

$$\Xi_s(\boldsymbol{n}; t) = \cos\frac{\omega t}{2} - \mathrm{i}\sqrt{3}\cos\theta \sin\frac{\omega t}{2}. \tag{12}$$

Consider generally now an orbital part of the following quadratic form:

$$H(\boldsymbol{u}, t) = \tfrac{1}{2}\boldsymbol{u}^t \mathsf{B}(t)\boldsymbol{u} + \boldsymbol{u}^t c(t) + d(t) \tag{13}$$

where $\mathsf{B}(t)$ is a 6×6 symmetric matrix, $\boldsymbol{c}(t)$ is a six-dimensional vector and $d(t)$ is a real function. It is possible to find a solution of Hamilton's equations of motion in the form

$$\boldsymbol{u}(t, t_0) = \boldsymbol{\Sigma}(t, t_0)\boldsymbol{u}_0 + \boldsymbol{a}(t, t_0) \tag{14}$$

where $\boldsymbol{\Sigma}(t, t_0)$ is a 6×6 matrix and $\boldsymbol{u}_0$ are the initial conditions, so that $\boldsymbol{\Sigma}(t_0, t_0) = \mathsf{I}$ and $\boldsymbol{a}(t_0, t_0) = \mathbf{0}$. Hamilton's equations give us the following differential equations for $\boldsymbol{\Sigma}$ and $\boldsymbol{a}$

$$\dot{\boldsymbol{\Sigma}}(t, t_0) = \mathsf{JB}(t)\boldsymbol{\Sigma}(t, t_0) \qquad \dot{\boldsymbol{a}}(t, t_0) = \mathsf{JB}(t)\boldsymbol{a}(t, t_0) + \mathsf{J}c(t). \tag{15}$$

Once we have solved (15), we can obtain the Moyal propagator by means of the following formula [11]

$$\Xi_H(\boldsymbol{u}; t) = F(t, t_0) \exp\left[\mathrm{i}(\boldsymbol{u}^t \mathsf{G}\boldsymbol{u} + \boldsymbol{u}^t \boldsymbol{k})\right] \tag{16}$$

with

$$\begin{aligned} \mathsf{G} &= \mathsf{J}(\boldsymbol{\Sigma} + \mathsf{I})^{-1}(\boldsymbol{\Sigma} - \mathsf{I}) = \mathsf{J} - 2\mathsf{J}(\boldsymbol{\Sigma} + \mathsf{I})^{-1} \\ \boldsymbol{k} &= 2\mathsf{J}(\boldsymbol{\Sigma} + \mathsf{I})^{-1}\boldsymbol{a} = (\mathsf{J} - \mathsf{G})\boldsymbol{a}. \end{aligned} \tag{17}$$

The function $F(t, t_0)$ can be shown to be

$$F(t,t_0) = \frac{\exp[\mathrm{i}\beta(t)]}{\sqrt{\det((\boldsymbol{\Sigma}(t,t_0)+\mathsf{I})/2)}} \tag{18}$$

with

$$\beta(t) = \int_{t_0}^{t} [\tfrac{1}{2}c^t(s)\mathbf{J}k(s) + \tfrac{1}{8}k^t(s)\mathbf{J}\mathsf{B}(s)\mathbf{J}k(s) - d(s)]\mathrm{d}s. \tag{19}$$

We summarize the forthcoming comptutation as follows.

(i) Solve the system of differential equations (15). We take $t_0 = 0$. The second system has the trivial solution $\boldsymbol{a}(t) \equiv \mathbf{0}$, which simplifies the calculations. The first system has solution

$$\boldsymbol{\Sigma}(t) = \mathrm{e}^{t\mathbf{J}\mathsf{B}} = \sum_{n=0}^{\infty} \frac{t^n}{n!}(\mathbf{J}\mathsf{B})^n \tag{20}$$

due to the fact that B is time-independent. Using (3) and the orbital part in (10) we obtain after some algebra

$$\boldsymbol{\Sigma}(t) = \begin{pmatrix} 1 & -\sin\omega t & 0 & \sin\omega t/m\omega & -(1-\cos\omega t)/m\omega & 0 \\ 0 & \cos\omega t & 0 & (1-\cos\omega t)/m\omega & \sin\omega t/m\omega & 0 \\ 0 & 0 & 1 & 0 & 0 & t/m \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -m\omega\sin\omega t & 0 & \sin\omega t & \cos\omega t & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{21}$$

(ii) Evaluate (17). In this case $k(t) \equiv \mathbf{0}$, due to the fact that $\boldsymbol{a}(t) \equiv \mathbf{0}$. We obtain

$$\mathsf{G} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -m\omega\tan(\omega t/2) & 0 & \tan(\omega t/2) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tan(\omega t/2) & 0 & -(m\omega)^{-1}\tan(\omega t/2) & 0 & 0 \\ 0 & 0 & 0 & 0 & -(m\omega)^{-1}\tan(\omega t/2) & 0 \\ 0 & 0 & 0 & 0 & 0 & -t/2m \end{pmatrix}. \tag{22}$$

(iii) To know $F(t)$ we need the function $\beta(t)$ in (19), which is identically zero in this case. We then have $F(t) = (\cos\omega t/2)^{-1}$.

(iv) Collecting the results of (i), (ii) and (iii), we get

$$\Xi_h(\boldsymbol{u};t) = \frac{1}{\cos\omega t/2}\exp\left[-\frac{2\mathrm{i}}{\omega}\left(\tan\frac{\omega t}{2}\right)\left(\frac{p_1^2+p_2^2}{2m} + \frac{m\omega^2}{2}q_2^2 - \omega p_1 q_2\right)\right]\exp\left(-\mathrm{i}t\frac{p_3^2}{2m}\right). \tag{23}$$

We can see that the variables $(1, 2)$ play a different role from that of variable (3). This fact is due to the choice of direction for the magnetic field.

(v) The last step is to obtain the spectral projector and its support. The propagator is

$$\Xi_H(\boldsymbol{u},\boldsymbol{n};t) = \frac{1}{\cos\omega t/2}\exp\left\{-\frac{2\mathrm{i}}{\omega}h(1,2)\tan\frac{\omega t}{2} - \mathrm{i}t\frac{p_3^2}{2m}\right\} \times \frac{1}{2}\left[\exp\left(\frac{\mathrm{i}\omega t}{2}\right)(1-\sqrt{3}n_z) + \exp\left(-\frac{\mathrm{i}\omega t}{2}\right)(1+\sqrt{3}n_z)\right]. \tag{24}$$

By $h(1,2)$ we represent the part of the Hamiltonian which depends only on these variables. It appears explicitly written in (23).

Now, we evaluate the spectral projector:

$$\begin{aligned} P_H(\boldsymbol{u},\boldsymbol{n};E) &= \frac{1}{2\pi}\int_{\mathbb{R}}\Xi_H(\boldsymbol{u},\boldsymbol{n};t)\mathrm{e}^{\mathrm{i}tE}\,\mathrm{d}t \\ &= \frac{1-\sqrt{3}n_z}{4\pi}\int_{\mathbb{R}}\frac{\mathrm{d}t}{\cos\omega t/2}\exp\left\{\mathrm{i}t\left(E+\frac{\omega}{2}-\frac{p_3^2}{2m}\right)-\frac{2\mathrm{i}}{\omega}h(1,2)\tan\frac{\omega t}{2}\right\} \\ &\quad + \frac{1+\sqrt{3}n_z}{4\pi}\int_{\mathbb{R}}\frac{\mathrm{d}t}{\cos\omega t/2}\exp\left\{\mathrm{i}t\left(E-\frac{\omega}{2}-\frac{p_3^2}{2m}\right)-\frac{2\mathrm{i}}{\omega}h(1,2)\tan\frac{\omega t}{2}\right\}. \end{aligned} \tag{25}$$

To evaluate these integrals, we use the identity

$$\frac{1}{\cos y/2}\exp\left(-\frac{\mathrm{i}x}{2}\tan\frac{y}{2}\right) = \sum_{n=0}^{\infty}(-1)^n 2\exp(-x/2)L_n(x)\exp(-\mathrm{i}y(n+\tfrac{1}{2})) \tag{26}$$

which comes from the definition of Laguerre polynomials by means of their generating function.

From (26), we have

$$\begin{aligned} &\int_{\mathbb{R}}\frac{\mathrm{d}t}{\cos\omega t/2}\exp(\mathrm{i}\lambda t)\exp\left(-\frac{\mathrm{i}x}{2}\tan\frac{\omega t}{2}\right) \\ &\quad = \sum_{n=0}^{\infty}(-1)^n 2\exp(-x/2)L_n(x)\int_{\mathbb{R}}\exp(\mathrm{i}\lambda t - \mathrm{i}\omega t(n+\tfrac{1}{2}))\,\mathrm{d}t \\ &\quad = 4\pi\sum_{n=0}^{\infty}(-1)^n\exp(-x/2)L_n(x)\;\delta(\lambda-\omega(n+\tfrac{1}{2})). \end{aligned} \tag{27}$$

In our case

$$x = \frac{4h(1,2)}{\omega} \qquad \lambda = E \pm \frac{\omega}{2} - \frac{p_3^2}{2m}.$$

Therefore, we have

$$\begin{aligned} P_H(\boldsymbol{u},\boldsymbol{n};E) &= \sum_{n=0}^{\infty}(-1)^n\exp\left(\frac{-2h(1,2)}{\omega}\right)L_n\left(\frac{4h(1,2)}{\omega}\right) \\ &\quad \times\left\{(1-\sqrt{3}n_z)\delta\left(E-\left[\frac{p_3^2}{2m}+n\omega\right]\right)\right. \\ &\quad \left. + (1+\sqrt{3}n_z)\delta\left(E-\left[\frac{p_3^2}{2m}+\omega(n+1)\right]\right)\right\}. \end{aligned} \tag{28}$$

Here we see that the support on the variable E of this function is

$$E = \frac{p_3^2}{2m} + \omega(n + \tfrac{1}{2} \pm \tfrac{1}{2}). \tag{29}$$

This is the expected result, see formula (111.7) in Landau [12].

To finish the paper we are going to obtain the Wigner functions [7] corresponding to the eigenfunctions evaluated by Landau's formula (111.6) and we will see that they are, essentially, the coefficients of Dirac's deltas in (28). The spectral projector directly provides us with this information!

First of all, let us write the wavefunctions of Landau's formula (111.6) in a convenient form:

$$\begin{aligned}\psi(\boldsymbol{q}) &= \exp(\mathrm{i}(p_{10}q_1 + p_{30}q_3)) \exp\left[-\frac{|e|B}{2}\left(\frac{q_2 - p_{10}}{|e|B}\right)^2\right] H_n\left(\sqrt{|e|B}\left(q_2 - \frac{p_{10}}{|e|B}\right)\right) \\ &= \exp[\mathrm{i}(p_{10}q_1 + p_{30}q_3)] \exp\left[-\frac{m\omega}{2}\left(q_2 - \frac{p_{10}}{m\omega}\right)^2\right] H_n\left(\sqrt{m\omega}\left(q_2 - \frac{p_{10}}{m\omega}\right)\right).\end{aligned} \tag{30}$$

Here p_{10}, p_{30} are two fixed real numbers. $H_n(x)$ are Hermite polynomials. The Wigner function corresponding to this wavefunction is given by [7]

$$\begin{aligned}\mathcal{W}_\psi(\boldsymbol{q},\boldsymbol{p}) &= \left(\frac{1}{2\pi}\right)^3 \int_{\mathbb{R}^3} \psi^*\left(\boldsymbol{q} + \frac{\boldsymbol{v}}{2}\right) \psi\left(\boldsymbol{q} - \frac{\boldsymbol{v}}{2}\right) \mathrm{e}^{\mathrm{i}\boldsymbol{p}\boldsymbol{v}}\, \mathrm{d}\boldsymbol{v} \\ &= \left(\frac{1}{2\pi}\right)^3 \int_{\mathbb{R}^3} \exp\{\mathrm{i}[v_1(p_1 - p_{10}) + v_2 p_2 + v_3(p_3 - p_{30})]\} \\ &\quad \times \exp\left\{-m\omega\left[\left(q_2 - \frac{p_{10}}{m\omega}\right)^2 + \left(\frac{v_2}{2}\right)^2\right]\right\} \\ &\quad \times H_n\left(\sqrt{m\omega}\left(q_2 - \frac{p_{10}}{m\omega} + \frac{v_2}{2}\right)\right) \\ &\quad \times H_n\left(\sqrt{m\omega}\left(q_2 - \frac{p_{10}}{m\omega} - \frac{v_2}{2}\right)\right) \mathrm{d}v_1\, \mathrm{d}v_2\, \mathrm{d}v_3 \\ &= \frac{\mathrm{e}^{-a^2}}{\pi\sqrt{m\omega}} \delta(p_1 - p_{10})\delta(p_3 - p_{30}) \int_{\mathbb{R}} \mathrm{e}^{-x^2} \mathrm{e}^{\mathrm{i}bx}\, H_n(a + x) H_n(a - x)\, \mathrm{d}x.\end{aligned} \tag{31}$$

For the sake of simplicity we have written

$$a = \sqrt{m\omega}\left(q_2 - \frac{p_{10}}{m\omega}\right) \qquad x = \frac{\sqrt{m\omega}}{2} v_2 \qquad b = \frac{2p_2}{\sqrt{m\omega}} \qquad \alpha = a + \mathrm{i}\frac{b}{2}. \tag{32}$$

By using the parity properties of the Hermite polynomials, the integral in (31) could be written in the form

$$\begin{aligned}\mathcal{W}_\psi &= (-1)^n \frac{\mathrm{e}^{-(a^2 + (b/2)^2)}}{\pi\sqrt{m\omega}} \delta(p_1 - p_{10})\delta(p_3 - p_{30}) \int_{-\infty}^{\infty} \mathrm{e}^{-x^2} H_n(x + \alpha)\, H_n(x - \alpha^*)\, \mathrm{d}x \\ &= (-1)^n \frac{\mathrm{e}^{-(a^2 + (b/2)^2)}}{\pi\sqrt{m\omega}} \delta(p_1 - p_{10})\delta(p_3 - p_{30}) 2^n \sqrt{\pi} n!\, L_n(2\alpha\alpha^*).\end{aligned} \tag{33}$$

In the last step we have used the identity (7.377) of [13]. By $L_n(x)$ we represent the Laguerre polynomials. In order to finish this calculation, let us substitute formulae (32) in (33). We get

$$\mathcal{W}_\psi(q,p) = \frac{(-2)^n\, n!}{\sqrt{m\omega\pi}}\delta(p_1 - p_{10})\delta(p_3 - p_{30})\mathrm{e}^{-2h(1,2)/\omega} L_n\left(\frac{4h(1,2)}{\omega}\right). \tag{34}$$

The function $h(1,2)$ is the one given in formulae (23) and (24). As we anticipated, summing over p_{10}, p_{20} to reintroduce the degeneration, we recover, except for a constant factor, the coefficients of the spectral projector. The concrete form of the Wigner function depends on the gauge chosen.

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